UCRHEP-T429
January 2007

New Lepton Family Symmetry and Neutrino Tribimaximal Mixing

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Abstract

The newly proposed finite symmetry $\Sigma(81)$ is applied to the problem of neutrino tribimaximal mixing. The result is more satisfactory than those of previous models based on A_4 in that the use of auxiliary symmetries (or mechanisms) may be avoided. Deviations from the tribimaximal pattern are expected, but because of its basic structure, only $\tan^2\theta_{12}$ may differ significantly from 0.5 (say 0.45) with $\sin^2 2\theta_{23}$ remaining very close to one, and θ_{13} very nearly zero.

Based on present neutrino-oscillation data, the neutrino mixing matrix $U_{\alpha i}$ linking the charged leptons ($\alpha = e, \mu, \tau$) to the neutrino mass eigenstates ($i = 1, 2, 3$) is determined to a large extent [1]. In particular, a good approximate description is that of the so-called tribimaximal mixing of Harrison, Perkins, and Scott [2], i.e.

$$U_{\alpha i} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}. \tag{1}$$

Using the discrete lepton family symmetry group A_4 [3, 4], this pattern has been discussed in a number of recent papers with varying additional assumptions [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. In particular, auxiliary symmetries (or mechanisms) beyond A_4 are required to enforce the following conflicting alignment of vacuum expectation values: (1,1,1) for a **3** representation which couples to charged leptons, and (1,0,0) for a **3** representation which couples to neutrinos. As shown below, this problem may be alleviated if A_4 is replaced by another finite discrete symmetry $\Sigma(81)$, which was recently proposed [17].

Consider the basis (a_1, a_2, a_3) and the Z_3 transformation

$$a_1 \to a_2 \to a_3 \to a_1. \tag{2}$$

If this is supplemented with the Z_2 transformation

$$a_{1,2} \to -a_{1,2}, \quad a_3 \to a_3, \tag{3}$$

then the group generated is A_4, which is the symmetry group of the even permutation of 4 objects, and that of the perfect tetrahedron [18]. It is also a subgroup of $SU(3)$, denoted as $\Delta(12)$. If Eq. (2) is supplemented instead with another Z_3, i.e.

$$a_1 \to a_1, \quad a_2 \to \omega a_2, \quad a_3 \to \omega^2 a_3, \tag{4}$$

where $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$, then the group generated is $\Delta(27)$ [19, 20, 21], which is also a subgroup of $SU(3)$. If Eq. (4) is replaced with

$$a_1 \to \omega a_1, \quad a_{2,3} \to a_{2,3}, \tag{5}$$

then the group generated, call it $\Sigma(81)$, contains $\Delta(27)$. It is a subgroup of $U(3)$ but not $SU(3)$. It has 9 one-dimensional irreducible representations $\mathbf{1_i}(i = 1, ..., 9)$ and 8 three-dimensional ones $\mathbf{3_A}, \mathbf{\bar{3}_A}, \mathbf{3_B}, \mathbf{\bar{3}_B}, \mathbf{3_C}, \mathbf{\bar{3}_C}, \mathbf{3_D}, \mathbf{\bar{3}_D}$. Its 17×17 character table and the 81 matrices of its defining representation $\mathbf{3_A}$ are given in Ref. [17].

Consider the supersymmetric extension of the Standard Model with 3 lepton families. Under $\Sigma(81)$, let

$$L_i = (\nu_i, l_i) \sim \mathbf{3_A}, \quad l_i^c \sim \mathbf{1_{1,2,3}}, \quad \Phi = (\phi^0, \phi^-) \sim \mathbf{1_1}, \tag{6}$$

$$\sigma_i \sim \mathbf{3_A}, \quad \bar{\sigma}_i \sim \mathbf{\bar{3}_A}, \quad \chi_i \sim \mathbf{3_B}, \quad \bar{\chi}_i \sim \mathbf{\bar{3}_B}, \quad \xi = (\xi^{++}, \xi^+, \xi^0) \sim \mathbf{1_1}. \tag{7}$$

Using the multiplication rules given in the Appendix, the allowed quadrilinear Yukawa terms are $(L_1\bar{\sigma}_1 + L_2\bar{\sigma}_2 + L_3\bar{\sigma}_3)l_1^c\Phi$, $(L_1\bar{\sigma}_1 + \omega^2 L_2\bar{\sigma}_2 + \omega L_3\bar{\sigma}_3)l_2^c\Phi$, $(L_1\bar{\sigma}_1 + \omega L_2\bar{\sigma}_2 + \omega^2 L_3\bar{\sigma}_3)l_3^c\Phi$, $(L_1L_1\sigma_1 + L_2L_2\sigma_2 + L_3L_3\sigma_3)\xi$, and $(L_1L_2\chi_3 + L_2L_3\chi_1 + L_3L_1\chi_2)\xi$. As shown below, the singlet superfields σ_i, $\bar{\sigma}_i$, χ_i, $\bar{\chi}_i$ will acquire vacuum expectation values <u>without</u> breaking the supersymmetry. The desirable solutions (1,1,1) for σ_i, $\bar{\sigma}_i$, and (1,0,0) for χ_i and $\bar{\chi}_i$ may then be obtained in a natural symmetry limit, for which the mismatch between the charged-lepton and neutrino mass matrices will exhibit tribimaximal mixing.

The most general superpotential of the singlet superfields invariant under $\Sigma(81)$ is given by

$$\begin{aligned} W &= m_\sigma(\sigma_1\bar{\sigma}_1 + \sigma_2\bar{\sigma}_2 + \sigma_3\bar{\sigma}_3) + m_\chi(\chi_1\bar{\chi}_1 + \chi_2\bar{\chi}_2 + \chi_3\bar{\chi}_3) \\ &+ \frac{1}{3}f(\sigma_1^3 + \sigma_2^3 + \sigma_3^3) + \frac{1}{3}\bar{f}(\bar{\sigma}_1^3 + \bar{\sigma}_2^3 + \bar{\sigma}_3^3) + \frac{1}{3}h(\chi_1^3 + \chi_2^3 + \chi_3^3) + \frac{1}{3}\bar{h}(\bar{\chi}_1^3 + \bar{\chi}_2^3 + \bar{\chi}_3^3) \\ &+ \lambda(\chi_1\sigma_2\sigma_3 + \chi_2\sigma_3\sigma_1 + \chi_3\sigma_1\sigma_2) + \bar{\lambda}(\bar{\chi}_1\bar{\sigma}_2\bar{\sigma}_3 + \bar{\chi}_2\bar{\sigma}_3\bar{\sigma}_1 + \bar{\chi}_3\bar{\sigma}_1\bar{\sigma}_2). \end{aligned} \tag{8}$$

The resulting scalar potential has a supersymmetric minimum ($V = 0$) if

$$0 = m_\sigma\bar{\sigma}_1 + f\sigma_1^2 + \lambda(\chi_2\sigma_3 + \chi_3\sigma_2) = m_\sigma\sigma_1 + \bar{f}\bar{\sigma}_1^2 + \bar{\lambda}(\bar{\chi}_2\bar{\sigma}_3 + \bar{\chi}_3\bar{\sigma}_2), \tag{9}$$

$$0 = m_\sigma\bar{\sigma}_2 + f\sigma_2^2 + \lambda(\chi_3\sigma_1 + \chi_1\sigma_3) = m_\sigma\sigma_2 + \bar{f}\bar{\sigma}_2^2 + \bar{\lambda}(\bar{\chi}_3\bar{\sigma}_1 + \bar{\chi}_1\bar{\sigma}_3), \tag{10}$$

$$
\begin{aligned}
0 &= m_\sigma \bar\sigma_3 + f\sigma_3^2 + \lambda(\chi_1\sigma_2 + \chi_2\sigma_1) = m_\sigma\sigma_3 + \bar f \bar\sigma_3^2 + \bar\lambda(\bar\chi_1\bar\sigma_2 + \bar\chi_2\bar\sigma_1), && (11)\\
0 &= m_\chi \bar\chi_1 + h\chi_1^2 + \lambda\sigma_2\sigma_3 = m_\chi\chi_1 + \bar h\bar\chi_1^2 + \bar\lambda\bar\sigma_2\bar\sigma_3, && (12)\\
0 &= m_\chi \bar\chi_2 + h\chi_2^2 + \lambda\sigma_3\sigma_1 = m_\chi\chi_2 + \bar h\bar\chi_2^2 + \bar\lambda\bar\sigma_3\bar\sigma_1, && (13)\\
0 &= m_\chi \bar\chi_3 + h\chi_3^2 + \lambda\sigma_1\sigma_2 = m_\chi\chi_3 + \bar h\bar\chi_3^2 + \bar\lambda\bar\sigma_1\bar\sigma_2. && (14)
\end{aligned}
$$

In the limit $\lambda = \bar\lambda = 0$, the symmetry of W is enlarged to $\Sigma(81) \times \Sigma(81)$. Thus it is natural to expect $\lambda, \bar\lambda << f, \bar f, h, \bar h$, and as a first approximation, a possible solution of $V = 0$ is

$$
\begin{aligned}
&\langle\sigma_{1,2,3}\rangle_0 = -m_\sigma(f^2\bar f)^{-1/3}, \quad \langle\bar\sigma_{1,2,3}\rangle_0 = -m_\sigma(\bar f^2 f)^{-1/3}, && (15)\\
&\langle\chi_1\rangle_0 = -m_\chi(h^2\bar h)^{-1/3}, \quad \langle\bar\chi_1\rangle_0 = -m_\chi(\bar h^2 h)^{-1/3}, \quad \langle\chi_{2,3}\rangle_0 = \langle\bar\chi_{2,3}\rangle_0 = 0, && (16)
\end{aligned}
$$

where $\lambda = \bar\lambda = 0$ has been assumed. This results in the desirable Yukawa terms $(l_1 + l_2 + l_3)l_1^c\phi^0$, $(l_1 + \omega^2 l_2 + \omega l_3)l_2^c\phi^0$, $(l_1 + \omega l_2 + \omega^2 l_3)l_3^c\phi^0$, $(\nu_1\nu_1 + \nu_2\nu_2 + \nu_3\nu_3)\xi^0$, and $\nu_2\nu_3\xi^0$, leading to tribimaximal mixing [5]. Specifically

$$
\mathcal{M}_l = \begin{pmatrix} h_e & h_\mu & h_\tau \\ h_e & \omega^2 h_\mu & \omega h_\tau \\ h_e & \omega h_\mu & \omega^2 h_\tau \end{pmatrix} v = \frac{1}{\sqrt3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}\begin{pmatrix} h_e & 0 & 0 \\ 0 & h_\mu & 0 \\ 0 & 0 & h_\tau \end{pmatrix}\sqrt3 v, \tag{17}
$$

and [6, 10]

$$
\mathcal{M}_\nu = \begin{pmatrix} a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 1/\sqrt2 & 0 & i/\sqrt2 \\ 1/\sqrt2 & 0 & -i/\sqrt2 \end{pmatrix}\begin{pmatrix} a+d & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & -a+d \end{pmatrix}\begin{pmatrix} 0 & 1/\sqrt2 & 1/\sqrt2 \\ 1 & 0 & 0 \\ 0 & i/\sqrt2 & -i/\sqrt2 \end{pmatrix}, \tag{18}
$$

thereby leading to Eq. (1), i.e.

$$
\frac{1}{\sqrt3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 1/\sqrt2 & 0 & i/\sqrt2 \\ 1/\sqrt2 & 0 & -i/\sqrt2 \end{pmatrix} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}. \tag{19}
$$

This is thus another version of a successful derivation of tribimaximal mixing, but as in all previous such models, the predicted value of $\tan^2\theta_{12} = 0.5$ is not the central value of present experimental data: $\tan^2\theta_{12} = 0.45 \pm 0.05$. To obtain a deviation from $\tan^2\theta_{12} = 0.5$ in the context of $\Sigma(81)$ alone, consider now $\lambda, \bar{\lambda} \neq 0$ but small. In that case,

$$\frac{\langle\chi_{2,3}\rangle}{\langle\chi_1\rangle_0} \simeq \frac{\bar{\lambda}(h^2\bar{h})^{1/3}}{(f^2\bar{f})^{2/3}}\frac{m_\sigma^2}{m_\chi^2}, \quad \frac{\langle\bar{\chi}_{2,3}\rangle}{\langle\bar{\chi}_1\rangle_0} \simeq \frac{\lambda(\bar{h}^2h)^{1/3}}{(\bar{f}^2f)^{2/3}}\frac{m_\sigma^2}{m_\chi^2}, \tag{20}$$

$$\delta\langle\sigma_1\rangle \simeq 0, \quad \frac{\delta\langle\sigma_{2,3}\rangle}{\langle\sigma\rangle_0} \simeq -\frac{m_\chi}{3m_\sigma}\left[\frac{\bar{\lambda}f^{1/3}}{(\bar{h}^2h\bar{f})^{1/3}} + \frac{2\lambda\bar{f}^{1/3}}{(h^2\bar{h}f)^{1/3}}\right], \tag{21}$$

$$\delta\langle\bar{\sigma}_1\rangle \simeq 0, \quad \frac{\delta\langle\bar{\sigma}_{2,3}\rangle}{\langle\bar{\sigma}\rangle_0} \simeq -\frac{m_\chi}{3m_\sigma}\left[\frac{\lambda\bar{f}^{1/3}}{(h^2\bar{h}f)^{1/3}} + \frac{2\bar{\lambda}f^{1/3}}{(\bar{h}^2h\bar{f})^{1/3}}\right]. \tag{22}$$

Since $\langle\bar{\sigma}_1\rangle \neq \langle\bar{\sigma}_{2,3}\rangle$, the charged-lepton mass matrix is modified. Instead of Eq. (17), it is now of the form

$$\mathcal{M}_l = \begin{pmatrix} h_e v_1 & h_\mu v_1 & h_\tau v_1 \\ h_e v_2 & \omega^2 h_\mu v_2 & \omega h_\tau v_2 \\ h_e v_2 & \omega h_\mu v_2 & \omega^2 h_\tau v_2 \end{pmatrix}. \tag{23}$$

Using the phenomenological hierarchy $h_e << h_\mu << h_\tau$, it is easily shown [15], to first approximation, that the tribimaximal $U_{\alpha i}$ of Eq. (1) is multiplied on the left by

$$R = \begin{pmatrix} 1 & -r & -r \\ r & 1 & -r \\ r & r & 1 \end{pmatrix}, \quad r \simeq \frac{v_1 - v_2}{v_1 + 2v_2} \simeq -\frac{\delta\langle\bar{\sigma}_{2,3}\rangle}{3\langle\bar{\sigma}\rangle_0}. \tag{24}$$

The neutrino mass matrix of Eq. (18) is also changed, i.e.

$$\mathcal{M}_\nu = \begin{pmatrix} a & e & e \\ e & a+b & d \\ e & d & a+b \end{pmatrix}, \tag{25}$$

where $|b| << |a|$ and $|e| << |d|$. This leads to a correction of Eq. (1) on the right by the matrix

$$R' = \begin{pmatrix} 1 & -r' & 0 \\ r' & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad r' \simeq \frac{\sqrt{2}e}{d} \simeq \frac{\sqrt{2}\langle\chi_{2,3}\rangle}{\langle\chi_1\rangle_0}. \tag{26}$$

Hence the corrected mixing matrix is given by

$$U_{\alpha i} \simeq \begin{pmatrix} \sqrt{2/3}(1 + r + r'/\sqrt{2}) & \sqrt{1/3}(1 - 2r - \sqrt{2}r') & 0 \\ -\sqrt{1/6}(1 - 3r - \sqrt{2}r') & \sqrt{1/3}(1 + r'/\sqrt{2}) & -\sqrt{1/2}(1 + r) \\ -\sqrt{1/6}(1 - r - \sqrt{2}r') & \sqrt{1/3}(1 + 2r + r'/\sqrt{2}) & \sqrt{1/2}(1 - r) \end{pmatrix}. \tag{27}$$

Therefore,

$$\tan^2\theta_{12} \simeq \frac{1}{2} - 3(r + r'/\sqrt{2}), \quad \tan^2\theta_{23} \simeq 1 + 4r, \quad \theta_{13} \simeq 0. \tag{28}$$

For example, let $r = r' = 0.01$, then $\tan^2\theta_{12} \simeq 0.45$, whereas $\tan^2\theta_{23} \simeq 1.04$ which is equivalent to $\sin^2 2\theta_{23} \simeq 0.9996$. A better match to the data is thus obtained.

In conclusion, it has been shown in this paper that neutrino tribimaximal mixing is a natural limit in a supersymmetric model based on $\Sigma(81)$ alone. Whereas corrections are expected, they are such that only $\tan^2\theta_{12}$ may deviate significantly from 0.5 without affecting much the predictions $\sin^2 2\theta_{23} = 1$ and $\theta_{13} = 0$.

This work was supported in part by the U. S. Department of Energy under Grant No. DE-FG03-94ER40837.

Appendix

The 9 one-dimensional irreducible representations together with $\mathbf{3_D}$, $\mathbf{\bar{3}_D}$ behave as in $\Delta(27)$, i.e. [21]

$$\mathbf{3_D} \times \mathbf{3_D} = \mathbf{\bar{3}_D} + \mathbf{\bar{3}_D} + \mathbf{\bar{3}_D}, \quad \mathbf{3_D} \times \mathbf{\bar{3}_D} = \mathbf{1_{1,2,3}} + \mathbf{1_{4,5,6}} + \mathbf{1_{7,8,9}}. \tag{29}$$

The $\mathbf{3_A}, \mathbf{3_B}, \mathbf{3_C}$ representations are cyclically equivalent, as are their conjugates. Their multiplication rules are

$$\mathbf{3_A} \times \mathbf{\bar{3}_A} = \mathbf{3_B} \times \mathbf{\bar{3}_B} = \mathbf{3_C} \times \mathbf{\bar{3}_C} = \mathbf{1_{1,2,3}} + \mathbf{3_D} + \mathbf{\bar{3}_D}, \tag{30}$$

$$\mathbf{3_B} \times \mathbf{\bar{3}_A} = \mathbf{3_C} \times \mathbf{\bar{3}_B} = \mathbf{3_A} \times \mathbf{\bar{3}_C} = \mathbf{1_{4,5,6}} + \mathbf{3_D} + \mathbf{\bar{3}_D}, \tag{31}$$

$$\mathbf{3_C} \times \mathbf{\bar{3}_A} = \mathbf{3_A} \times \mathbf{\bar{3}_B} = \mathbf{3_B} \times \mathbf{\bar{3}_C} = \mathbf{1_{7,8,9}} + \mathbf{3_D} + \mathbf{\bar{3}_D}, \tag{32}$$

$$\mathbf{3_A} \times \mathbf{3_A} = \mathbf{3_B} \times \mathbf{3_C} = \mathbf{\bar{3}_A} + \mathbf{\bar{3}_B} + \mathbf{\bar{3}_B}, \tag{33}$$

$$\mathbf{3_B} \times \mathbf{3_B} = \mathbf{3_C} \times \mathbf{3_A} = \mathbf{\bar{3}_B} + \mathbf{\bar{3}_C} + \mathbf{\bar{3}_C}, \tag{34}$$

$$\mathbf{3_C} \times \mathbf{3_C} = \mathbf{3_A} \times \mathbf{3_B} = \mathbf{\bar{3}_C} + \mathbf{\bar{3}_A} + \mathbf{\bar{3}_A}. \tag{35}$$

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